The dependence of

$$[(m_a - m)/Nm]1/a^{2-n}L^{n+1} = \alpha(n)$$

on n is given in Table 1. With n = 1 (Newtonian solvent)

$$[(m_a - m)/Nm]1/aL^2 = (3/2)\pi,$$

which coincides with the results of [2].

LITERATURE CITED

1. R. B. Bird and H. R. Warner, "Kinetic theory and rheology of dumbbell suspensions with Brownian motion," in: Advances in Polymer Science (edited by H. J. Cantow et al.), Vol. 8, Springer-Verlag (1971), pp. 1-90.
2. E. Yu. Taran, "Rheological equation of state of dilute suspensions of rigid dumbbells with spheres at their ends," Prikl. Mekh., 11, (1975).
3. Yu. Tomita, "On the fundamental formula of non-Newtonian flow," Bull. JSME, 2, No. 7 (1959).
4. J. C. Slattery and R. B. Bird, "Non-Newtonian flow past a sphere," Chem. Eng. Sci., 16, Nos. 3/4 (1961).
5. G. C. Wallick, J. G. Savins, and D. R. Arterburn, "Tomita's solution for the motion of a sphere in a power-law fluid," Phys. Fluids, 5, No. 3 (1962).
6. M. L. Wasserman and J. C. Slattery, "Upper and lower bounds on the drag coefficient of a sphere in a power-model fluid," AIChE J., 10, No. 3 (1965).
7. G. Biardi, G. Antolini, F. Lesco, and M. Dente, "Applicazione numerica di un metodo o variazionale alla risoluzione di problema di moto puramente viscoso per fluidi nonnewtoniani," Atti Accad. Naz. Lincei Rend, Cl. Sci. Fis., Mat. Natur., 47, Nos. 3/4 (1970).
8. J. L. Ericksen, "Transversally isotropic fluids," Kolloid. Z., 173, No. 2, (1960).
9. R. S. Rivlin and J. L. Ericksen, "Stress—strain relations for isotropic materials," J. Rat. Mech. Anal., 4, No. 3 (1955).
10. E. Janke, F. Emde, and F. Lösch, Special Functions, Formulas, Curves, Tables [Russian translation], Izd. Nauka, Moscow (1968), p. 55.

HEAT TRANSFER WITH THE FLOW OF STRUCTURALLY VISCOUS MEDIA IN TUBES AND CHANNELS

T. Negmatov and P. V. Tsoi

UDC 536.25

In various branches of modern technology, wide use is made of so-called structurally viscous media, which, in their physical properties, differ considerably from ordinary Newtonian liquids. Structurally viscous media include high-polymer, colloidal, bulk, coarsely dispersed, and other systems, for which the Newton hypothesis of a linear dependence between the rate of deformation and the stress no longer holds. A nonlinear dependence between the stress and the gradient of the rate of flow is the most characteristic special feature of non-Newtonian liquids [1]; this dependence is frequently expressed by the Ostwald formula

$$\tau = k(dw/dr)^m. \tag{1}$$

For a laminar, hydrodynamically stabilized flow of anomalous liquids with an exponential rheological law (1), the field of the velocities in a round tube and a plane-parallel channel is expressed by the formula

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$$w(\xi)/\langle w\rangle = \{[(2+\Gamma)m+1]/(m+1)\}[1-\xi^{(m+1)/m}], \tag{2}$$

where $\Gamma = 0$, $\xi = (y/h)\,\mathrm{sign}\, y$; $-h \leqslant y \leqslant h$ for a slit-type channel; $\Gamma = 1$; $0 \leqslant \xi = r/R \leqslant 1$ for a round tube; and $\langle w\rangle = w_{av}$ is the average velocity over a cross section of the flow of liquid.

We assume that the outer surface of the tube (channel) is washed by a medium with a temperature $T_{med} = \varphi(X)$ and that heat transfer with the external medium takes place in accordance with Newton's law. The semibounded tube has a preceding connected hydrodynamic stabilization section; the liquid with a temperature of T_0 enters the active zone of the tube with a fully developed stabilized velocity (2). Then, for a thermally thin tube (channel), determination of the temperature field in a flow of non-Newtonian liquid, with the generally known assumptions, reduces to solution of the following boundary-value problem [2]:

$$\{[(2+\Gamma)m+1]/(m+1)\}\left[1-\xi^{(m+1)/m}\right]\partial T/\partial X = \frac{1}{\xi^{\Gamma}}\frac{\partial}{\partial \xi}\left(\xi^{\Gamma}\frac{\partial T}{\partial \xi}\right) + \frac{q(\xi, X)R^2}{\lambda_L}; \tag{3}$$

$$T(\xi, X)|_{X\leqslant 0} = T_0, \quad \{\partial T/\partial \xi = \mathrm{Bi}[\varphi(X) - T(\xi, X)]\}_{\xi=1}, \tag{4}$$

where

$$X = (x/R)\cdot 1/\mathrm{Pe}, \quad \mathrm{Pe} = w_{av}R/a, \quad \mathrm{Bi} = \alpha R/\lambda_L \quad 0 \leqslant x < \infty;$$

λ_L and a are the coefficients of thermal conductivity and thermal diffusivity of the liquid; α is the coefficient of heat transfer between the surface of the tube and the external medium. Thus, the quantity Bi differs from the generally adopted Biot number, introduced in the theory of thermal conductivity.

A more exact mathematical model of a theoretical investigation of internal problems of convective heat transfer with the flow of a heat-transfer medium in a tube reduces to determination of the temperature fields in the flow and over the thickness of the wall of the tube, i.e., it reduces to the solution of so-called conjugated problems [3]. Problem (3), (4), as a conjugated problem, is stated in the following manner. It is required to find the distribution of the temperature $T_1(\xi, X)$ over the thickness of the wall of the tube, satisfying the equation of thermal conductivity and the boundary conditions (4) at the outer surface, where $\mathrm{Bi} = \alpha R/\lambda$; λ is the coefficient of thermal conductivity of the wall of the tube. At the outer surface, the solution $T_1(\xi, X)$ must satisfy the conditions of conjugation with the temperature in the flow of liquid $T_2(\xi, X)$;

$$[T_1(\xi, X)]_{\xi=1+0} = [T_2(\xi, X)]_{\xi=1-0}, \quad (-\lambda\partial T_1/\partial\xi)_{\xi=1+0} = (-\lambda_L\partial T_2/\partial\xi)_{\xi=1-0}. \tag{5}$$

Theoretical methods for solving conjugated problems are bound up with surmounting complex mathematical transformations, and the solutions known in the literature are expressed by cumbersome functional dependences and are not very suitable for practical calculations.

In the classical statement of problems of convective heat transfer, formulated by Graetz [2], the temperature conditions are usually given at the outer surface of the tube and are limited to determination of the temperature field in the flow of liquid. The solutions of such problems are suitable for the investigation of heat transfer in tubes with a thermally thin wall.

By the introduction of an unknown function of the temperature distribution $\varphi(X)$ at the liquid–wall interface:

$$(T_1)_{\xi=1-0} = (T_2)_{\xi=1+0} = \varphi(X)$$

the solution of the conjugated problem can be reduced to solution of the Graetz problem in the flow of liquid and the problem of thermal conductivity over the thickness of the wall of the tube. To find $\varphi(X)$, we obtain an integral equation from the second condition of (5).

Consequently, the representations of the temperature in the flow of liquid $T_2(\xi, X)$ and at the wall of the tube $T_1(\xi, X)$ are simple and sufficiently exact expressions which permit finding effective solutions of conjugated problems. One such method, proposed in [4], will be used for the solution of Eq. (3) with the boundary conditions (4). Obviously, with $\mathrm{Bi} = \infty$, the problem stated coincides with a generalized problem of the Graetz type.

We introduce the Laplace transform

$$T^*(\xi, s) = \int_0^\infty T(\xi, X)\exp(-sX)\,dX;$$

then, with respect to the transform $T^*(\xi, s)$, from (3), (4), we obtain

$$\frac{d}{d\xi}\left(\xi^\Gamma \frac{dT^*}{d\xi}\right) - [sT^*(\xi, s) - T_0]\,w(\xi)\,\xi^\Gamma + \frac{q^*(\xi, s)\,R^2\xi^\Gamma}{\lambda L} = 0; \tag{6}$$

$$\{dT^*/d\xi + \mathrm{Bi}\,T^*(\xi, s)\}_{\xi=1} = \mathrm{Bi}\,\varphi^*(s). \tag{7}$$

Determination of the exact solution of the boundary-value problem (6), (7) and a transition to the region of inverse transforms present certain mathematical difficulties, and the final results are expressed by complicated analytical dependences. Therefore, the determination of the temperature field in a simple form, even at the price of decreasing its accuracy, is of practical importance for engineering thermophysics.

For the solution of internal problems of convective heat transfer, a rather effective method is the orthogonal projection of an unconnected boundary-value problem of the kind (6), (7) in a functional space with a finite number of dimensions [4].

We postulate that the distribution of the internal sources of heat evolution is stabilized along the flow of the liquid, i.e.,

$$\lim_{X\to\infty} q(\xi, X) = \lim_{s\to 0} sq^*(\xi, s) = q(\xi). \tag{8}$$

We seek an approximate solution $T^*(\xi, s)$, satisfying the boundary condition (7), in the family of a linear composition of the type

$$T_n^*(\xi, s) = \varphi^*(s) + \sum_{k=1}^{n} a_k^*(s)\,\psi_k(\xi, \mathrm{Bi}), \tag{9}$$

where the coordinate functions $\psi_k(\xi, \mathrm{Bi})$ are linearly independent and satisfy the conditions

$$\{d\psi_k/d\xi + \mathrm{Bi}\,\psi_k(\xi, \mathrm{Bi})\}_{\xi=1} = 0.$$

The choice of the first coordinate function $\psi_1(\xi, \mathrm{Bi})$ depends on the analytical expression of the limiting function $q(\xi)$ in the relationship (8). For example, with $q(\xi) = q_v = \mathrm{const}$, solution (9) is brought to the form

$$T_n^*(\xi, s) = \varphi^*(s) + a_1^*(s)\left(\frac{\mathrm{Bi}+2}{\mathrm{Bi}} - \xi^2\right) + \sum_{k=2}^{n} a_k^*(s)(1-\xi^2)^k.$$

In the case $q(\xi) = q_v(1 - \xi^2)$, including the condition

$$q(\xi, X) = q_v(1-\xi^2)[1 - \exp(-\mathrm{Pd}X)]$$

we must take as the first coordinate function for a round tube ($\Gamma = 1$) [4]

$$\psi_1(\xi, \mathrm{Bi}) = \xi^4 - 4\xi^2 + (3\mathrm{Bi} + 4)/\mathrm{Bi}.$$

With such a choice of the system of coordinate functions, the approximate solution (9) in the region of inverse transforms with an increase in X approaches the exact solution.

The coordinates of the transform $a_k^*(s)$ are projections of the vector $T_n^*(\xi, s) - \varphi^*(s)$ on the coordinate axes in a functional space with a finite number of dimensions and are found from the requirement of the orthogonality of the residual of equation (6) with $T^* = T_n^*$ with respect to all the coordinate functions $\psi_j(\xi)$:

$$\int_0^1 \left\{\frac{d}{d\xi}\left(\xi^\Gamma \frac{dT_n^*}{d\xi}\right) - [sT_n^*(\xi, s) - T_0]\,w(\xi)\,\xi^\Gamma + \frac{q^* R^2 \xi^\Gamma}{\lambda L}\right\}\psi_j(\xi)\,d\xi = 0. \tag{10}$$

System (10), after integration with respect to ξ, is brought to the form

$$\sum_{k=1}^{n}\{A_{jk}+B_{jk}s\}\,a_k^*(s)=[T_0-s\varphi^*(s)]\,C_j+\frac{R^2}{\lambda_L}D_j^*(s) \tag{11}$$

$$(j=1,2,\ldots,n),$$

where

$$A_{jk}=A_{kj}=-\int_0^1\frac{d}{d\xi}\left(\xi^\Gamma\frac{d\psi_k}{d\xi}\right)\psi_j(\xi)\,d\xi>0,$$

$$B_{jk}=B_{kj}=\int_0^1 w(\xi)\,\psi_j\psi_k\xi^\Gamma d\xi>0,$$

$$C_j=\int_0^1 w(\xi)\,\psi_j(\xi)\,\xi^\Gamma d\xi,\quad D_j^*(s)=\int_0^1 q^*(\xi,s)\,\psi_j(\xi)\,\xi^\Gamma d\xi.$$

Determining the coefficients $a_k^*(s)$ from system (11), and going over to the region of inverse transforms using the formula

$$a_k(X)=\sum_{j=1}^{n}\int_0^X\varphi_1(\alpha)\left\{\sum_{i=1}^{n}\frac{\Delta_{jk}(s_i)}{\Delta'(s_i)}\exp[s_i(X-\alpha)]\right\}d\alpha+\frac{R^2}{\lambda_L}\sum_{j=1}^{n}\int_0^X D_j(\alpha)\left\{\sum_{i=1}^{n}\frac{\Delta_{jk}(s_i)}{\Delta'(s_i)}\exp[s_i(X-\alpha)]\right\}d\alpha$$

in relationship (9), we obtain the solution of the starting problem. Here $s_i < 0$ are the simple roots of the equation $\Delta(s) = 0$; $\Delta(s) = |A + sB|$ is the principal determinant of the system; Δ_{jk} is its algebraic complement;

$$\varphi_1(X)=T_0-s\varphi^*(s);\quad D_j(X)=D_j^*(s).$$

Calculation of the coefficients $a_k(X)$ in the third and following approximations ($n \geqslant 3$) is carried out effectively using an electronic computer. Here, for simplification, all the calculations must be made with fixed values of the rheological parameter m and the Bi number. In particular, the relative excess temperature in the flow of liquid inside a round tube ($\Gamma = 1$) with $\varphi(X) = T_{med} = \text{const}$, $q(\xi, X) = 0$, and $m = 1/3$ (a pseudoplastic) is brought to the form

$$\Theta(\xi,X,\mathrm{Bi})=\frac{T(\xi,X)-T_{\mathrm{med}}}{T_0-T_{\mathrm{med}}}=\sum_{k=1}^{n}\varphi_k(\xi,\mathrm{Bi})\exp[s_k(\mathrm{Bi})X]. \tag{12}$$

Curves of the change in the local Nusselt number

$$\mathrm{Nu}=N(X)=-2(d\Theta/\partial\xi)_{\xi=1}/\langle\Theta\rangle,$$

calculated using solution (12), are given in Fig. 1.

The results of calculation of the temperature in the flow of liquid and the local Nusselt number for a round tube and a slit-type channel ($\Gamma = 0;\ 1$) with $m = 1$, $q = 0$, and $\mathrm{Bi} = \infty$ gave good agreement with known investigations of other authors.

The numerical realization of system (11) with concrete conditions of singularity permits investigating a broad circle of problems of convective heat transfer for anomalous media, taking account of given distribution functions of the internal sources of heat evolution, the heat of friction, and various laws of the change in the temperature of the external medium. We consider below problems of heat transfer in a round tube with a linear rise of the temperature of the external medium along the flow and an investigation of the temperature field in a slit-type channel, due to the dissipation of energy.

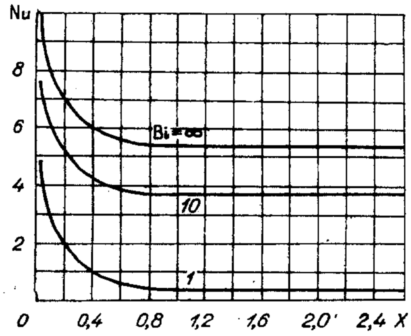

Fig. 1

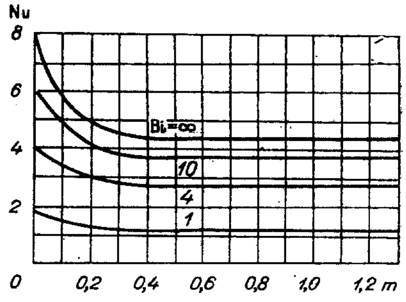

Fig. 2

In Eqs. (3), (4) we set

$$\Gamma = 1,\ q = 0,\ T_{med} = \varphi(X) = T_0 + \Delta T^* X,\ \Delta T^* = \Delta T\,\mathrm{Pe}\,R.$$

We first find the solution far from the inlet to the tube. We seek the stabilized field of the temperature in the form

$$T(\xi,\ X,\ \mathrm{Bi}) = T_0 + \Delta T^* X + T_1(\xi,\ \mathrm{Bi}). \tag{13}$$

We substitute the value of (13) into Eq. (3). Integrating the equation obtained with zero boundary conditions of the third kind, we find $T_1(\xi,\ \mathrm{Bi})$. The relative excess temperature far from the inlet to the tube is expressed by the formula

$$(14)\quad \Theta(\xi, X, \mathrm{Bi}) = \frac{T - T_0}{\Delta T^*} = X - \frac{3m+1}{m+1}\left[\frac{(5m^2+6m+1)\,\mathrm{Bi} + 2\,(m+1)\,(3m+1)}{4\,(3m+1)^2\,\mathrm{Bi}} - \xi^2/4 + \left(\frac{m}{3m+1}\right)^2 \xi^{\frac{3m+1}{m}}\right]. \tag{14}$$

Differentiating with respect to ξ, we obtain

$$-2(\partial\Theta/\partial\xi)_{\xi=1} = 1,$$

i.e., the temperature gradient at the wall with a sufficiently large value of X does not depend on m and Bi. The minimal Nusselt number, determined using solution (14), has the form

$$\mathrm{Nu}_{\min} = \frac{-2(\partial\Theta/\partial\xi)_{\xi=1}}{\langle\Theta\rangle - \Theta_c} = \frac{8\,(5m+1)\,(3m+1)\,(m+1)\,\mathrm{Bi}}{(31m^3+43m^2+13m+1)\,\mathrm{Bi} + 4\,(m+1)(3m+1)\,(5m+1)}. \tag{15}$$

If, in (14), (15), we set m = 1 and Bi = ∞, then, for a normal Newtonian liquid, we obtain the known solutions [2]:

$$\Theta(\xi,\ X) = X - (1/8)(3 - 4\xi^2 + \xi^4),\ \mathrm{Nu}_{\min} = 48/11 = 4.364.$$

The change in $\mathrm{Nu}_{\min}$ for Bi = 1, 4, 10, ∞ is given in Fig. 2.

Preliminary investigations of the temperature field in the stabilized section permit constructing solutions for the whole zone of the tube with respect to such a system of coordinate functions, with which, for a fixed value of n, we obtain the best approximation.

This solution is found in the form

$$T_n^*(\xi, s, \mathrm{Bi}) = T_0/s + \Delta T^*/s^2 + a_1^*(s)\left[\frac{(5m^2+6m+1)\,\mathrm{Bi} + 2\,(m+1)\,(3m+1)\,(5m+1)}{4\,(3m+1)\,\mathrm{Bi}} - \right.$$
$$\left. - \frac{\xi^2}{4} + \left(\frac{m}{3m+1}\right)^2 \xi^{\frac{3m+1}{m}}\right] + \sum_{k=2}^{n} a_k^*(s)\,(1-\xi^2)^k. \tag{16}$$

On the basis of the calculations made and the properties of a Laplace transform, we find

$$\lim_{s\to 0} s a_1^*(s) = \lim_{X\to\infty} a^1(X) = -\frac{3m+1}{m+1},\ \lim_{s\to 0} s a_k^*(s) = \lim_{X\to\infty} a_k(X) = 0,\ k \geqslant 2.$$

Consequently, the approximate solution (16) in the region of inverse transforms with an increase in the value of X tends toward an exact asymptotic solution (14).

A realization of solution (16) for a Newtonian liquid ($m = 1$) with boundary conditions of the first kind ($Bi = \infty$) is given in [4]. With a finite value of the Bi number, the exactness of the calculation improves and the error decreases with a decrease in the Bi number.

A characteristic special feature of the flow of a number of rheological media, for example, molten plastics, is the relatively great value of their effective viscosity. This leads to a considerable dissipation of mechanical energy into thermal with the flow of such liquids in tubes and channels.

In Eqs. (3), (4) we set $q(\xi, X)R^2/\lambda_L = \mu/\lambda\cdot(dw/d\xi)^2$, $\Gamma = 0$, $T_{med} = T_0 = \text{const}$; then the solution of the boundary-value problem (5), (6), with an averaged viscosity coefficient ($\mu = \text{const}$), is found in the form

$$T_n^*(\xi, X, \mathrm{Bi}) = T_{med}/s + a_1^*(s)\left[\frac{m(\mathrm{Bi}+2)+2}{m\,\mathrm{Bi}} - \xi^{\frac{2(m+1)}{m}}\right] + \sum_{k=2}^{n}(1-\xi^4)^k a_k^*(s).$$

For the coefficient $a_1^*(s)$, solving the truncated system (11) of the first order, we obtain

$$a_1^*(s) = \frac{\mu w_{av}^2}{2\lambda_L}\frac{(2m+1)^2}{(m+1)(m+2)}\left\{\frac{1}{s} - \left[\frac{mM_1\,\mathrm{Bi}^2 + M_2\,\mathrm{Bi}}{(M_3\,\mathrm{Bi}^2 + M_4\,\mathrm{Bi} + M_1)(3m+4)} + s\right]^{-1}\right\},$$

where

$$M_1(m) = 1440m^6 + 7272m^5 + 15236m^4 + 16952m^3 + 10564m^2 + 3496m + 480;$$

$$M_2(m) = 4(m+1)^2(3m+4)(3m+2)(4m+3)(5m+4)(6m+5);$$

$$M_3(m) = 384m^6 + 1024m^5 + 1026m^4 + 462m^3 + 80m^2;$$

$$M_4(m) = 1440m^6 + 5592m^5 + 8652m^4 + 6680m^3 + 2580m^2 + 400m.$$

The relative excess temperature inside a plane-parallel channel, in the first approximation, is equal to

$$\Theta(\xi, X, \mathrm{Bi}) = \frac{T - T_0}{\mu w_{av}^2/\lambda_L} = \frac{(2m+1)^2}{2(m+1)(m+2)}\left\{1 - \exp\left[-\frac{(mM_1\,\mathrm{Bi}^2 + M_2\,\mathrm{Bi})X}{(M_3\,\mathrm{Bi}^2 + M_4\,\mathrm{Bi} + M_1)(3m+4)}\right]\right\}\left[\frac{m(\mathrm{Bi}+2)+2}{m\,\mathrm{Bi}} - \xi^{\frac{2(m+1)}{m}}\right]. \quad (17)$$

Far from the inlet to the tube, where $\exp\left[-\frac{(m\,\mathrm{Bi}^2M_1 + M_2\,\mathrm{Bi})X}{(M_3\,\mathrm{Bi}^2 + M_4\,\mathrm{Bi} + M_1)(3m+4)}\right] \approx 0$, expression (17) coincides with the exact asymptotic solution. Specifically, with $Bi = \infty$, from (17) we find

$$T(\xi, X) = T_0 + \frac{3}{4}\frac{\mu w_{av}^2}{\lambda_L}[1 - \exp(-2.661X)](1 - \xi^4)$$

and, as $X \to \infty$, in the limit we obtain the Schlichting solution [5]. The proposed method determines without complication the solution with a variable viscosity coefficient, depending on ξ.

The method of the combined application of integral transforms in an orthogonal projection to internal problems of convective heat transfer can be successfully realized with any given analytical expression of the stabilized profile of the velocity $w(\xi)$. In cases where the information on the distribution of the velocity was obtained from experimental data or by integratinn of the equation of hydrodynamics, the coefficients in system (11) can be found by approximate integration with respect to discrete points.

The authors of [6] proposed new analytical dependences for the relative velocity of the structurally viscous flow of a liquid in a round tube or a flat slit in the form

$$\frac{w(\xi)}{\langle w\rangle}=2\frac{(1-\xi^2)+\frac{2}{3}\frac{\Theta}{\varphi_0}\tau_w(1-\xi^3)}{1+\frac{4}{5}\frac{\Theta}{\varphi_0}\tau_w}.$$

These data permit investigating heat transfer taking account of the new complex $\Theta\tau_w/\varphi_0$.

LITERATURE CITED

1. S. S. Kutateladze, E. M. Khabakhpasheva, V. B. Lemberksii, and V. I. Popov, "Questions of the hydrodynamics and heat transfer of structurally viscous media," in: Heat and Mass Transfer in Newtonian Liquids [in Russian], Izd. Énergiya, Moscow (1968).
2. B. S. Petukhov, Heat Transfer and Resistance with the Laminar Flow of a Liquid in Tubes [in Russian], Izd. Énergiya, Moscow (1967).
3. A. V. Lykov, A. A. Aleksashenko, and V. A. Aleksashenko, Conjugated Problems in Convective Heat Transfer [in Russian], Izd. BGU, Minsk (1971).
4. P. V. Tsoi, Methods for Calculating Individual Problems in Heat and Mass Transfer [in Russian], Izd. Énergiya, Moscow (1971).
5. H. Graeber, S. Erk, and U. Grigul, Principles of Heat-Transfer Science [Russian translation], Izd. Inostr. Lit., Moscow (1958).
6. S. S. Kutateladze, V. I. Popov, and E. M. Khabakhpasheva, "The hydrodynamics of liquids with a variable viscosity," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1966).

SELF-SIMILAR PROBLEMS OF TURBULENT MIXING AT THE INTERFACE OF COMPRESSIBLE GASES*

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INTRODUCTION

It is well known that the interface of liquids or gases located in a field of gravity breaks down if a heavy substance is located above a light one. An analogous picture arises in the absence of a gravity field, if the light substance accelerates the heavy one. The theory of turbulent mixing and the corresponding self-similar solution for incompressible liquids are constructed in [1].

For some self-similar problems in gasdynamics there arise conditions leading to turbulent mixing. In the present work, solutions are constructed taking account of turbulent mixing. The article discusses the problem of the motion of two originally cold gases, in one of which there is given a rising evolution of energy, varying in accordance with a power or exponential law. In a self-similar solution at an interface, moving with an acceleration, there appears a discontinuity of the density: a shock wave enters the cold gas, leaving behind it a high (at the interface, infinite) density, while a rarefaction wave is propagated into the energy-evolving gas. The interface is obviously unstable, i.e., the light substance accelerates the heavy one. For this problem, a solution is constructed taking account of turbulent mixing.

The article considers the motion of a gas under the action of an applied pressure, rising either stepwise or exponentially. The surface of the gas, to which the pressure is applied, is free. Such a piston can be obtained where, in a vacuum, the pressure is given (for example, of light). A free surface is unstable with respect to small perturbations. In distinction from known self-similar solutions [2, 3], the solution obtained with turbulent mixing

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